

INTERLINKAGE, LIMITED LIABILITY AND STRATEGIC INTERACTION

by

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ABSTRACT:

The literature on interlinkage is inconclusive regarding the strict superiority of this arrangement. We analyze a landlord and a moneylender as two players making non-cooperative decisions regarding the terms of their respective contracts with a tenant. In the sequential game where the landlord moves first and the tenant has limited liability, we demonstrate that there exist circumstances in which interlinkage is superior, even with nonlinear loan contracts, a result that carries over when there is moral hazard. The incorporation of risk aversion yields strict superiority in general. The main result is unaffected by changes in the seniority of claims, but is sensitive to changes in the order of moves: limited liability ceases to ensure the strict superiority of interlinked contracts if the principal who provides the variable factor of production moves first, even if he has junior claims to the output.

1. Introduction

The central idea in the literature on interlinking is a simple one: if one principal acquires all instruments, then with transferable utility, he can do at least as well as any set of principals among whom all instruments are distributed and who act non-cooperatively. To take the well-worn example of a landlord, a moneylender and a tenant, let the landlord have access to finance on the same terms as the moneylender. Then he always has the option of offering an interlinked tenancy-cum-credit contract whose terms are identical to those of the contracts that are the outcome of the game involving the landlord and moneylender as separate principals. The question that arises at once is: can the landlord do strictly better? It is far from a foregone conclusion that he can. To give three very different instances in which he cannot, consider first the case where there is no uncertainty and the tenant is subject to unlimited liability (Bhadhuri, 1973). Then, as Newbery (1975) shows, a first-best allocation in which the landlord enjoys the entire surplus can be achieved through the use of a single instrument. Secondly, in a uncertain world in which enough variables are observable, Ray and Sengupta (1989) show that, given a particular set of conditions, a landlord can do just as well by shunning moneylending provided he can impose sufficiently nonlinear contracts. Thirdly, if the setting is one of adverse selection, in which tenants' discount rates are their private information, then only the more impatient among them will take up interlinked contracts, which also involve departures from the first-best allocation (Banerji, 1995).

The purpose of this paper is to show that there are potential gains from coordination through interlinking even in circumstances that seem, at first sight, rather unpromising. Thus, in establishing the

main result, we rule out any moral hazard by denying the tenant any choices other than whether to accept a contract.¹ We also assume that the landlord can costlessly observe the output from the tenancy, so that depending on the other information available to him, he can specify either state- or output-contingent payments. Although we do not confine our attention to the case where the tenant is risk neutral, we begin with it in order to sharpen the argument; for in this case, risk-sharing has no role to play. All this contrasts with the standard explanations of interlinkage which rely on moral hazard or the absence of nonlinear pricing (see Bell, 1988, for discussion).

In our model, the occurrence of interlinkage is basically driven by the assumption of limited liability. Indeed the model may be viewed as demonstrating the power of limited liability. While this assumption has not been used too widely in the theoretical literature on agrarian organization (for its use in the context of tenancy, see Kotwal, 1985; Basu, 1992), it is empirically very robust in backward agrarian economies. In the event of a crop failure due to a drought or a flood, it is the norm rather than the exception for the principal to forego some of the contractual claims that he or she may have on the agent (Atchi Reddy, 1996; Jones, 1962).

When, as is plausible, the realized output can be so small as to make it impossible for the tenant to meet his contractual obligations, it is the provisions for default that have a central place, especially when there are two principals who act non-cooperatively. For the contractual terms cannot be specified without a clear rule to settle what are competing claims in the event of a default. In the present setting, it is natural to assume that the landlord has first claim on the tenant's output. As

¹ It will be recalled that moral hazard is the cause of interlinkage in the model of Braverman and Stiglitz (1981).

a rule, he lives in the same village, he may have some say in when the crop is harvested, and if the rent depends on the level of output, the crop is usually harvested and threshed under his watchful eye. The moneylender, however, is often an outsider, who is far worse placed to exercise such a claim. Turning to the question of the sequence of moves, note that a landless and assetless tenant will typically not get a loan unless he has a tenancy. Consequently, we assume that in the absence of interlinkage, the landlord will move first, and that there will be a sequential (Stackelberg) game, with the landlord as leader. For completeness, however, the robustness of our results to the relaxation of these assumptions involving the sequence of moves and the seniority of claims is examined in section 6.

The plan of the paper is as follows. The basic model, in which all players are risk neutral and there is effectively full observability, is set out in section 2. Under interlinking, the landlord's optimum therefore entails a first-best allocation, a well known result that is proved in section 3. The sequential game with both principals is analyzed in section 4, where it is shown that a first-best allocation is possible if and only if the landlord is able to specify a fixed rent independently of the state of nature without violating the tenant's individual rationality constraint. This will be feasible only if output in the worst state of nature is sufficiently high; otherwise, interlinkage will be advantageous. Here, it should be noted that in our model, the landlord can write contracts which are state-contingent but not ones which are contingent on the amount of credit taken. This latter is a crucial assumption, but we feel it is quite reasonable to assume that the volume of credit taken is not observable to third parties outside the borrower and the lender. In section 5, we examine the question of risk-sharing when the tenant is risk averse. Interlinking permits the attainment of a first-best allocation, in which the landlord bears all

risks. It is shown, however, that in the absence of interlinkage, if the effort of the tenant is not variable, there exists no Nash-Stackelberg equilibrium in which the landlord absorbs all risks. Consequently, no equilibrium of the sequential game can be allocatively efficient in this case. We then extend our analysis to include moral hazard, which arises when the tenant's effort is variable and it is prohibitively costly to observe or monitor it. Our conclusion here is identical to that of section 4: there are gains to interlinking if the landlord does not charge a uniform rent. As a second generalization, section 6 examines the effects of changes in the order of play, and of the transfer of seniority of claims to the moneylender. The paper concludes with a brief discussion.

2. The Model

The tenant-cultivator is assumed to apply his endowment of an indivisible unit of labor to a plot of land. The output produced is stochastic, with two possible outcomes: a high yield (denoted by y_H), and a bad harvest, or low yield (denoted by y_L). The probability of the good outcome is an increasing, strictly concave and twice differentiable function of K , $p(K)$, where K is the amount of investment in non-labor inputs, with $p'(K) > 0$ and $p''(K) < 0$. We additionally assume that $p(K)$ satisfies both Inada conditions, i.e. $\lim_{K \rightarrow 0} p'(K) = +\infty$ and $\lim_{K \rightarrow +\infty} p'(K) = 0$, to ensure the existence of interior solutions.

There is only one landowner in the village from whom a plot can be rented. It is assumed that he can observe the outcome, and so make the rent contingent thereon. Let β_i be the rent in state i ($i = H, L$). Lacking any funds of his own, the tenant must borrow to finance K , the use of which the lender can observe. If the landlord has access to funds, he may offer credit on terms which are

interlinked with the rental agreement. If he does not, or chooses not to lend, then the source of the loan is a professional moneylender. The opportunity cost of funds is the same for the landlord and the moneylender, and is a constant $(1 + m)$ per dollar loaned.

All three agents are assumed to be risk neutral. The reservation income of the tenant is \bar{y} , which is assumed to be perfectly certain. The investment in inputs is made essential for participation in cultivation by assuming that $p(0) = 0$ and $\bar{y} > y_L$.

If the output produced by the tenant is less than the combined amounts of the rent and credit obligations, there will be a shortfall in repayment of dues. The question arises as to how this shortfall will be shared. As indicated in the Introduction, we assume that the landlord has the "first rights" to the harvest, due to either the nature of property rights in the countryside, or his greater power and proximity to the tenancy, and thus eliminate the possibility of situations which involve bargaining strategies. With this assumption, it is easy to see that we do not sacrifice any generality by confining our attention to those rental contracts where $\beta_i \leq y_i$, for $i = L, H$; for demanding a state-contingent rent that is larger than the output in that state is pointless whatever be the landlord's bargaining power. It is also assumed that the monopoly power of the lender (or the landlord-cum-lender) over the tenant enables him to offer "all-or-nothing" loans which specify both the rate of interest and the amount of the loan.

We analyze the landlord's optimum under interlinkage first, and then compare it to the outcomes when the landlord and the moneylender make separate and independent decisions. From this comparison, we establish the conditions under which the interlinking of the land and the credit

markets leads to allocations that cannot be reproduced when the landlord and the moneylender play non-cooperatively. In the latter case, we assume that the tenant contracts with the moneylender after he has accepted the terms of the rent as specified by the landlord. This is a realistic assumption, which naturally makes the landlord a first mover, and the lender a follower, in the sequential game that is developed in section 4.

3. The Optimal Interlinked Contract

When the landlord offers interlinked loans, the terms of payment for both the land and the loan can be subsumed under a contract that appropriately specifies the state-contingent dues β_L and β_H , together with the amount of funds advanced to the tenant. The optimal values of these variables can be derived from the following program:

$$\begin{aligned} & \text{Maximize } [p(K)\beta_H + (1 - p(K))\beta_L - (1 + m)K] \\ & \beta_L, \beta_H, K \end{aligned} \tag{1}$$

subject to the tenant's participation constraint

$$p(K)(y_H - \beta_H) + (1 - p(K))(y_L - \beta_L) \geq \bar{y}. \tag{2}$$

It is assumed that there exists a solution involving non-negative values of β_H , β_L , and K such that the landlord's expected income is positive.² It is clear that (2) will be binding at the optimum, β_H and β_L

² We assume that the landlord and the moneylender do not supplement the earnings of the tenant, and earn negative payoffs, in any state of nature. For simplicity, we regard y_H and y_L suitably high to ensure the existence

being lump-sum transfer instruments. Hence, the objective function of the landlord can be expressed as

$$p(K)y_H + (1 - p(K))y_L - (1 + m)K - \bar{y}.$$

The assumptions on $\pi(\cdot)$ ensure that there is an interior solution with respect to K , given by the associated first-order condition

$$p'(K)[y_H - y_L] = 1 + m, \quad (3)$$

Let the optimal value of K be denoted by K^o , where it should be noted that K^o is the amount of investment that maximizes the expected output net of the opportunity cost of producing it, and is independent of the amount of rent. Substituting for K^o in (2), and recalling that (2) is strictly binding in the optimal solution and that $0 \leq \beta_i \leq y_i$, it is seen that with $\bar{y} > 0$, any pair (β_H, β_L) in the non-negative region of the line segment so described will, together with K^o , induce a fully efficient allocation of resources.³

4. A Separate Moneylender

We now consider the alternative situation where the tenant borrows the necessary funds

of interior solutions involving non-negative β_i and R_i , for $i=L, H$, in all cases.

³ We assume that \bar{y} is sufficiently low to make non-negative rents feasible. Note that if $\bar{y} = 0$, the (unique) optimal rents are given by $\beta_i = y_i$.

from a separate, monopolistic moneylender following the rental agreement with the landlord. In the sequential game to be analyzed, the lender moves second, and chooses the size of the loan, and the state-contingent terms of repayment R_H and R_L , as the best responses to (β_H, β_L) chosen by the landlord, so as to extract any remaining surplus from the tenant. The moneylender's program can be represented as

$$\text{Maximize } \{\pi(K)\min(R_H, y_H - \beta_H) + (1 - \pi(K))\min(R_L, y_L - \beta_L) - (1 + m)K\} \quad (4)$$

$$\{R_H, R_L, K\}$$

subject to the tenant's participation constraint, which takes the form

$$p(K)(y_H - \beta_H) + (1 - p(K))(y_L - \beta_L) - \{\pi(K)\min(R_H, y_H - \beta_H) + (1 - \pi(K))\min(R_L, y_L - \beta_L)\} \geq \bar{y} \quad (5)$$

Noting that the participation constraint of the tenant will bind at the optimum, inspection of (4) and (5) reveals that the problem of the moneylender reduces to maximizing the residual surplus after the claims of the other parties have been satisfied. That is, he chooses K in order to

$$\text{Maximize } \{[\pi(K)(y_H - \beta_H) + (1 - \pi(K))(y_L - \beta_L) - (1 + m)K] - \bar{y}\} \quad (6)$$

$$K$$

The associated first-order condition is

$$\pi'(K)[(y_H - y_L) - (\beta_H - \beta_L)] = 1 + m. \quad (7)$$

Let K^m represent the optimal choice of the moneylender. K^m will be the solution to (7) as long as

$$\pi(K^m)(y_H - \beta_H) + (1 - \pi(K^m))(y_L - \beta_L) - (1 + m)K^m \geq \bar{y},$$

i.e., the lender's resulting payoff (subject to the satisfaction of the tenant's participation constraint) is non-negative. Otherwise, $K^m = 0$.⁴ Provided that β_H and β_L are not so high as to render lending activities unprofitable, it is clear from (7) that K^m will depend on the terms of the rental contract, being a decreasing function of $\Delta\beta \equiv \beta_H - \beta_L$. As is evident from (3) and (7), $K^m \neq K^o$, except in the case where the rent is independent of the state of nature (i.e., $\beta_H = \beta_L$).

Now, not only is a uniform rent both necessary and sufficient to induce the lender to advance a loan of K^o , but it will also be optimal from the landlord's point of view if it can be chosen such that the moneylender and the tenant obtain exactly their reservation alternatives $(1+m)K^o$ and \bar{y} respectively. This is the case if there exists a $\beta^o \in (0, y_L]$ that satisfies

$$p(K^o)y_H + (1 - p(K^o))y_L - (1 + m)K^o - \beta^o - \bar{y} = 0. \quad (8)$$

The lender's best response is to choose R_H and R_L such that

$$p(K^o)[\min\{R_H, y_H - \beta^o\}] + (1 - p(K^o))[\min\{R_L, y_L - \beta^o\}] = (1 + m)K^o.$$

Such an outcome, if it exists, is fully efficient, so that there will be no gains from interlinking in this

⁴ This is so in the sense of the reservation payoff of the moneylender (net of the opportunity cost of funds) being zero.

case.⁵

If, however, the β^o that solves (8) has the property that $\beta^o > y_L$, such a uniform rent is infeasible, and the landlord's optimum involves rents that differ across the states of nature. As demonstrated in the appendix, it will be optimal for the landlord to appropriate the entire output when the yield is low (i.e., $\beta_L = y_L$), and β_H will be strictly greater than β_L . It follows at once from (7) that $K^m < K^o$, so that the outcome in the sequential game is inefficient (in the sense of production efficiency) and there will be gains from interlinkage. This is likely in practice, for droughts, floods and pestilence ensure that the value of y_L is small.

A special case, where it is always in the landlord's interest to offer an interlinked deal, occurs if $y_L = 0$, that is, the bad outcome is associated with no output. This case yields our result with particular transparency. Since $\beta_L = R_L = 0$, we may denote β_H by β , R_H by R and y_H by y without risk of confusion.

Once β is known, the moneylender's problem is to choose R and K so as to maximize $[p(K) \min \{R, y - \beta\} - (1 + m)K]$ such that $p(K)(y - \beta - R) \geq \bar{y}$. Since $\bar{y} \geq 0$, the constraint requires $y - \beta > R$. Hence, the maximand can be simplified accordingly and we get the first-order condition

$$p'(K) = \frac{1+m}{y-\beta}$$

Since efficiency requires $p'(K) = \frac{1+m}{y}$, inefficiency occurs whenever $\beta > 0$.

⁵ A higher uniform rent is infeasible, given that the participation of the lender is essential for production by virtue of the assumption that $\pi(0) = 0$.

Given that the landlord moves first, his problem is to choose β so as to maximize $p(K)\beta$, keeping in mind that K is implicitly defined by $p'(K) = \frac{1+m}{y-b}$. It is immediately evident that the chosen value of β exceeds zero. It follows that if the landlord displaces the moneylender by offering interlinked contracts to his tenant, he will do better.

The results of the preceding analysis are summarized in the following proposition:

Proposition 1: *Given the assumptions of Section 2, the sequential game between the landlord and the moneylender, where the landlord is the first mover and has “first rights” to the tenant’s harvest, yields the following outcome:*

- (i) $\beta_L = \min\{\beta^o, y_L\}$, where β^o is defined by (8).
- (ii) *If β^o is strictly greater than y_L , the nature of the tenant’s “limited liability” makes it optimal for the landlord to charge non-uniform rents, with*

$$\beta_H > \beta_L = y_L.$$

Interlinkage results in a higher expected yield, and a higher payoff to the landlord, in this case.

Proof: The proof of (i) is provided in the appendix. To show that $\beta_H > \beta_L = y_L$ when $\beta^o > y_L$, consider the effect of an increase in β_H on the landlord’s expected payoff. Let

$$O(K; \beta_H, \beta_L) = p(K)\beta_H + [1 - p(K)]\beta_L \quad (9)$$

Then,

$$O_K(K; \beta_H, \beta_L) = p'(K)(\beta_H - \beta_L) \frac{\partial K}{\partial \beta_H} + p(K) \quad (10)$$

from which it follows that $O_K(K; y_L, y_L) > 0$. By part (i), $\beta^o > y_L$ implies that $\beta_L = y_L$. Clearly, in this case, a small increase in β_H from the uniform rent of y_L , while not violating the participation constraints in the ensuing subgame, would result in an expected payoff which is strictly greater than y_L . Thus, the optimal rents are non-uniform, with β_H strictly greater than β_L . As discussed earlier, this results in $K^m < K^o$, which proves the rest of part (ii). //

It is important to stress that if the equilibrium in the sequential game involves non-uniform rents, the participation constraint of the moneylender need not necessarily bind. In the situation corresponding to part (ii) of Proposition 1, let (β_H^{\max}, y_L) be the pair of state-contingent rents such that, with K being given by (7), the participation constraint of the lender is binding. As long as β_H is strictly less than β_H^{\max} , increases in β_H above y_L decrease the amount of investment.⁶ Clearly, it is possible for β_H^* , the interior solution to $\frac{\partial \Omega}{\partial \beta_H} = 0$ to be strictly less than β_H^{\max} , in which case the moneylender chooses β_H^* as the optimal rent, and the moneylender's equilibrium payoff will be strictly positive.

⁶ In this case, there is an interior solution to the moneylender's program. His optimal choice of K is given by (7), from which it is easy to derive $\frac{\partial K}{\partial \beta_H} = \frac{p'(K)}{p''(K)(y_H - \beta_H)} < 0$.

Equations (3) and (7) reflect the fundamental difference between the incentives of the landlord and the moneylender with respect to advancing credit to the tenant. Since, under interlinkage, the landlord receives the entire output (and therefore any increase therein) net of \bar{y} , the L.H.S. of (3) represents the change in both the expected output and the landlord's payoff that results from an increment in K . If β_H is strictly greater than β_L , it follows from (7) that the tenant will obtain a smaller loan in the absence of interlinkage. This is because the independent lender does not get the entire increase in the expected yield. In fact, with the lender receiving the residual surplus *after* the rent has been appropriated by the landowner, an increase in K , while raising $p(K)$, also increases the "leakage" (in the form of the higher expected rent) from the residual that is available to the lender (over and above the reservation income of the tenant). As evident from (7), an increase in $\Delta\beta$ increases the leakage to the landlord, and reduces the optimal size of the moneylender's loan.

Observe that while we allow the landlord and the moneylender to write state-contingent contracts, we do not allow the former to write a contract contingent on the amount borrowed by the peasant. If we did allow such contracts, the landlord would specify two functions $\beta_H(K)$ and $\beta_L(K)$, which denote the state-and -loan contingent rents, such that

$$\beta_H(K^0) = y_H - \frac{?_H}{p(K^0)}$$

and

$$\beta_L(K^o) = y_L - \frac{\varphi_L}{1-p(K^o)}$$

where φ_H and φ_L are non-negative amounts that satisfy $\varphi_H + \varphi_L = (1+m)K^o$. For all other K , the landlord punishes the lender by charging a high enough rent (one which extracts all the output, say). This makes any amount of credit, other than the one that is optimal from the landlord's point of view, infeasible. On the other hand, the participation constraints of the moneylender and the tenant bind with a loan of size K^o . Thus, $\beta_i(K)$ as specified above, together with $R_i = (1+m)K^o$ for $i = L, H$, and $K = K^o$, constitute a sequential equilibrium which produces a first-best outcome even in the absence of interlinkage.

Viewed in this manner, our model may be interpreted as one that demonstrates interlinkage to be a consequence of the landlord's inability to make the rents contingent on the volume of loan when there is a separate moneylender. In this connection, we would like to argue that the volume of loan is typically unobservable. In collecting field data, it is well known that credit information is very hard to get, since it is such an intangible transaction. Moreover, in our model the value of K cannot be deduced from output, since output is unaffected by K . This point carries over to cases where K affects the volume of output, but there are other unobservable inputs (such as the tenant's effort) as well.

To conclude this section, it should be evident from (4) - (6) that if the lender is unable to observe the state of nature, and so must stipulate a uniform interest rate, then all of the above argument goes through as before. The tenant's limited liability simply implies that the lender may be

unable to collect the entire amount of the principal plus the interest in the bad state.

5. Risk-Sharing and Moral Hazard

If the tenant is risk averse, or his effort is prohibitively costly to monitor, then the sharing of risk and the provision of incentives, respectively, become central considerations in the determination of the contractual terms. Since the state-contingent payments cease, in general, to operate as lump-sum transfer instruments in this case, one might expect coordination over instruments through interlinkage to offer advantages over independent deals under a wider range of conditions than those described in Proposition 1. We begin by demonstrating that this indeed so in the case of a risk-averse tenant, even when his effort is not variable.

The tenant is assumed to have preferences over lotteries that can be represented by the von Neumann-Morgenstern utility function $u(x)$ for a risk-averse agent, where x denotes his realized income. Thus, the participation constraint becomes

$$p(K)u(y_H - \beta_H) + (1 - p(K))u(y_L - \beta_L) \geq u(\bar{y}). \quad (11)$$

Suppose, as in Section 3, the landlord is in a position to impose an interlinked contract. Replacing (2) by (11) in the landlord's problem when there is interlinkage yields the same first-order condition with respect to K , equation (3), as before. Those associated with β_i ($i = H, L$) yield $y_H - \beta_H = y_L - \beta_L$, that is, we have the standard result that the tenant receives a fixed payment \bar{y} , with the

landlord bearing all risk, a result that holds for any value of K . The allocation is fully efficient, as expected.

Matters are otherwise in the absence of interlinking. We can, indeed, go further than in Proposition 1.

Proposition 2: *If the tenant is risk averse, then in any equilibrium of the sequential game in which an independent moneylender moves second, the tenant will bear some risk. Hence, the associated allocation is always less efficient than that under interlinkage.*

Proof: Since the tenant's only means of repaying a loan is his income from cultivation, he will bear no risk if and only if

$$y_H - \beta_H = y_L - \beta_L > R_H = R_L \quad (12)$$

Suppose, therefore, that the landlord pays the tenant the fixed sum w . If the tenant is to bear no risk, the loan contract must take the form of a fixed repayment R that is feasible in both states of nature.

Thus, the lender's problem is

$$\begin{aligned} &\text{Maximize } [R - (1 + m)K] \\ &\{R, K | w\} \end{aligned} \quad (13)$$

subject to the participation constraint

$$w - R \geq \bar{y}. \quad (14)$$

If (14) is satisfied, then so is (12), with $w = y_H - \beta_H = y_L - \beta_L$. It is clear that there can be no equilibrium in which the lender participates unless $w > \bar{y}$. If $w > \bar{y}$, however, the moneylender's problem has the solution $K = 0$ and $R = w - \bar{y}$. Since the lender moves second, there is nothing the landlord can do to avoid such an outcome. Given the assumptions $\pi(0) = 0$ and $\bar{y} > y_L$, there will, moreover, be a positive lower limit on K such that the landlord finds it profitable to offer a tenancy. It follows that the sequential game will not possess an equilibrium if the landlord absorbs all the risks of production. If there does exist an equilibrium, therefore, its associated allocation of resources will be inefficient.

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It should be emphasized that in this setting, there will be with gains from the interlinking of contracts even without an appeal to the infeasibility of uniform rental payments when the bad outcome is a sufficiently miserable level of output, as is necessary when the tenant is risk-neutral (see Proposition 1).

To complete this section, we introduce moral hazard by assuming that observing and monitoring the tenant's effort, e , is now prohibitively costly. We initially continue with the assumptions of the indivisibility of e , and a production technology that requires that land and labor be employed in fixed proportions. Thus, the tenant's choice is confined to the two alternatives of

applying one indivisible unit of effort, or none at all, to the given unit of land. Our results are then extended to the case where the tenant may choose among all non-negative levels of e .

Specifically, we suppose that effort, as well as K , now has a positive effect on the probability of producing the higher level of output that is represented in the following manner

$$p(K, e) = \begin{cases} p(K) & \text{if } e=1 \\ 0 & \text{if } e=0 \end{cases} \quad (15)$$

where $\pi(K)$ has the same properties as in the previous section. Let x_i be the tenant's income in state

i. For simplicity of exposition, the tenant's utility function is assumed to be separable in x_i and e , and is represented as

$$V(x_i, e) = u(x_i) - e$$

where $u(x_i)$ is concave and twice differentiable in its argument. The reservation utility of the tenant is \bar{u} ($= u(\bar{y})$). After entering into all the necessary contractual agreements, the tenant chooses $e \in \{0, 1\}$ to

$$\text{maximize } \{p(K, e)u(x_H) + (1 - p(K, e))u(x_L) - e\}$$

provided that, with this optimal choice, his expected utility from participation is no less than \bar{u} .

Since he is the player who moves last of all, his state-contingent payoffs are the residual amounts that are left after the claims of the decision-maker in the previous stage of the game have been satisfied.

Irrespective of the identity of the penultimate player, the x_i 's that induce unit effort from the tenant must satisfy the following participation and incentive compatibility conditions

$$p(K)u(x_H) + (1 - p(K))u(x_L) - 1 = \bar{u} \quad (16a)$$

$$p(K)u(x_H) + (1 - p(K))u(x_L) - 1 = u(x_L) \quad (16b)$$

Following fairly standard arguments, it is easy to see that, provided $e = 1$ is desirable for the penultimate player, (16a) and (16b) will bind at his optimum, whereupon the solution values, x_H^* and x_L^* , satisfy $u(x_H^*) = \bar{u} + [\pi(K)]^{-1}$, and $u(x_L^*) = \bar{u}$.

The landlord-cum-moneylender will choose β_H , β_L and K , such that $\beta_i = y_i - x_i^*$ for $i = H, L$, and K maximizes

$$\pi(K)[y_H - x_H^*] + (1 - \pi(K))[y_L - x_L^*] - (1 + m)K$$

Keeping in mind that x_H depends on K , the first-order condition for an interior solution is

$$\pi'(K)[(y_H - y_L) - (x_H^* - x_L^*)] + \frac{p'(K)}{p(K)u'(x_H^*)} - (1 + m) = 0 \quad (17)$$

An important special case is one where the tenant is risk neutral, so that $V(\cdot)$ specializes to $V(x_i, e) = x_i - e$. The fact that (16b) is binding in the optimal solution then yields

$$\pi(K) (x_H^* - x_L^*) = 1$$

Consequently, (17) becomes identical to the corresponding first-order condition (3) in Section 3 which yields K^o as the optimal interlinked loan. With risk aversion, however, the optimal K that is derived from (17) will be strictly less than K^o : the compromise between risk-sharing and incentives takes a toll on efficiency.

The absence of interlinkage causes a further deterioration of allocative efficiency, unless the landlord is able to extract all surplus by charging a uniform rent. To show this, let K^* satisfy (17), and let β^* be such that

$$\pi(K^*)[y_H - x_H^*] + (1 - \pi(K^*))[y_L - x_L^*] - \beta^* - (1 + m)K^* = 0. \quad (18)$$

If β^* is strictly greater than y_L , then following the procedure of the previous section, it is easy to show that the landlord will find it optimal to charge $\beta_H > \beta_L$. This distorts the returns to the moneylender, net of the payments of x_H^* and x_L^* , and results in the latter providing an amount of funds lower than K^* . Observe that if the tenant is risk neutral, the incentive problem can be solved without reference to risk-sharing, so that we are back in the world of sections 3 and 4, where Proposition 1 holds.

For a brief demonstration of the robustness of our results when the tenant may choose any $e \in [0, +\infty)$, we proceed in a manner similar to Grossman and Hart (1983). That is, we suppose that the penultimate player first determines his minimum cost (in terms of the expected amount of residual

output that the contract leaves to the tenant) of inducing a particular level of effort from the tenant for any given K . This yields the cost function $C(e, K)$. He then finds the levels of induced effort and the size of the loan that maximize his expected payoff, net of the total costs $(1 + m)K$ and $C(e, K)$. Accordingly, the penultimate player chooses x_H and x_L , given K , to

$$\text{Minimize } \{p(K, e)x_H + (1 - p(K, e))x_L\} \quad (19)$$

subject to the tenant's participation constraint

$$p(K, e)u(x_H) + (1 - p(K, e))u(x_L) - e \geq \bar{u} \quad (20)$$

and her incentive compatibility constraint

$$e \in \underset{e'}{\text{argmax}} \{p(K, e')u(x_H) + (1 - p(K, e'))u(x_L) - e'\} \quad (21)$$

Under the assumptions of strict concavity of $u(\cdot)$ and $p(\cdot)$ with respect to their arguments, the minimization program (19) - (21) will yield $x_i^*(e, K)$ as the unique optimal solution, when

$$C(e, K) = p(K, e)x_H^*(e, K) + (1 - p(K, e))x_L^*(e, K)$$

The cost function $C(e, K)$ will be the same whether the player concerned be the landlord (with interlinkage) or the moneylender (as in the sequential game of Section 4). The difference lies in the nature of the benefit function of the penultimate player (i.e. his payoff net of $(1+m)K$ but gross of $C(e, K)$). In the case of an interlinked contract, this player is the landlord, who provides the loan and appropriates the surplus from the tenant. His benefit function is

$$B^{\ell}(e, K) = p(e, K)y_H + (1 - p(e, K))y_L - (1 + m)K \quad (22)$$

and he can be regarded as choosing e and K to maximize $B^{\ell}(e, K) - C(e, K)$. Let e^0 and K^0 represent the amount of inputs that are optimal from the landlord's point of view under interlinkage. Once these are determined, his optimal state-contingent rents follow from the corresponding values of $x_i^* = x_i(e^0, K^0)$, and from the relationship $x_i = y_i - \beta_i$ for $i = L, H$.

The procedure for uncovering the equilibrium decisions in the sequential game is similar, except that, in this case, it is the moneylender who determines the optimal e and K by maximizing the difference between $B^m(e, K; \beta_H, \beta_L)$ and $C(e, K)$, where his benefit function is

$$B^m(e, K; \beta_H, \beta_L) = p(e, K)(y_H - \beta_H) + (1 - p(e, K))(y_L - \beta_L) - (1 + m)K \quad (23)$$

Note that the difference between (22) and (23) is the expected rent. In fact, we can express (23) as

$$B^m(e, K; \beta_H, \beta_L) = B^{\ell}(e, K) - [p(e, K)\beta_H + (1 - p(e, K))\beta_L] \quad (24)$$

Once again, it is evident from (24) that if the subgame perfect equilibrium of the sequential game does not involve uniform rents, then a marginal increment in K , by increasing expected rents, will

result in leakages (of the same nature as described in section 4) from the incremental payoff accruing to the moneylender. Consequently, the marginal benefit from increasing the i -th input, given the level of the j -th input ($i, j = e, K$, with $i \neq j$), will be less for the moneylender than for the landlord, i.e. $B_i^l(e, K) > B_i^m(e, K)$ if $\beta_H \neq \beta_L$. Suppose there exist interior solutions to the maximization programs under interlinkage and in the sequential game, that uniform rents are not optimal, and that the second-order (sufficiency) are satisfied. Then, it is easy to show, using simple comparative statics, that interlinkage results in higher levels of e , K and expected output than the non-interlinked case. If the uniform rent that appropriates the entire surplus at the input levels e^o and K^o is feasible, it is easy to see that it will be the optimal choice of the landlord in the subgame perfect equilibrium of the sequential game. Interlinkage will not generate any additional advantage in this case.

Thus, the incorporation of moral hazard causes no change in the fundamental conclusion of section 4. In fact, this is true even if the agent is risk neutral. In this case, following the same procedure as before, it can, once again, be easily demonstrated that, if the limited liability condition renders the optimal uniform rent infeasible, interlinked contracts will be strictly superior in terms of production efficiency.

6. Extensions

The results of the previous section were established within the framework of a sequential game where the landlord was the first mover, and possessed senior claims to output. As mentioned in the Introduction, we regard this formulation to be the most appropriate reflection of reality, since

(i) a landless and assetless tenant will be considered creditworthy by informal sector moneylenders only after he comes into possession of the tenancy contract, and (ii) the landlord's proximity to the tenant gives him the advantage of superior accessibility to the output. Nevertheless, to complete the argument, we now examine how robust our results are to changes in these two assumptions. We also extend our analysis to allow the landlord to possess a variable instrument of control, and examine the consequences of such multiplicity of instruments on allocative efficiency. As it turns out, all these changes introduce important qualifications to our previous results.

6.1 *Changes in the Sequence of Decisions and Seniority of Claims*

For the sake of brevity, we avoid a complete and formal exposition of the games that involve alternative stipulations of seniority, together with different order of moves. Consider first the alternative scenario where the moneylender moves first, and has the "first rights" to the harvest. It is easy to see that the loan contract will specify R_H , R_L and K^0 such that K^0 satisfies (3), and R_H and R_L satisfy

$$\pi(K^0)(y_H - R_H) + (1 - \pi(K^0))(y_L - R_L) = \bar{\beta} + \bar{y} \quad (25)$$

where $\bar{\beta}$ represents the landlord's reservation income from the next best use of his plot. The outcome is first best, and interlinkage loses its superiority in this case.

Transference of seniority of claims from the first to the second mover introduces new complications. Irrespective of his identity, the player who moves first can now receive payoffs that differ from the state-contingent claims originally specified in his contract. To start with, suppose that

the moneylender follows the landlord in the sequence of decisions. With the former possessing the seniority of claims, the actual payoff received by the landlord in state i will be given by $\min\{\beta_i, y_i - R_i\}$, for $i = L, H$. Furthermore, for the equilibrium outcome of this game to be first best, the landlord must receive uniform payoffs across the states of nature. Otherwise, as obvious from the previous analysis, the moneylender will not find it optimal to offer K^0 . The condition that y_L be sufficiently large to enable the landlord to charge the uniform rent β^0 , where β^0 satisfies (8), while still necessary, is, however, no longer sufficient to ensure that K^0 constitutes the best response of the moneylender. For a first-best outcome, β^0 must, additionally, satisfy *both* the following conditions

$$y_H - \beta^0 < \frac{\bar{y}}{p(K^0)} \quad (26)$$

$$y_L - \beta^0 < \frac{\bar{y}}{1-p(K^0)} \quad (27)$$

and this implies that maintaining allocative efficiency is now more demanding than it was in section 4. Violation of *either* (26) *or* (27) implies that, even if $\beta^0 = y_L$, the equilibrium payoffs of the landlord are non-uniform across states of nature, in which case, the lender no longer finds it optimal to provide K^0 , the amount of funds that corresponds to the first-best outcome.

To see this, suppose, to the contrary, that there exists an equilibrium of this sequential game where the landlord receives the uniform rent (or payoff) $\beta (= y_L)$, following which the lender offers $\{K^0, R_H^0, R_L^0\}$ as the credit contract, where

$$y_i - \beta = R_i^0,$$

$$\pi(K^0)(y_H - R_H^0) + (1 - \pi(K^0))(y_L - R_L^0) - \beta = \bar{y}$$

and the magnitude of β^0 is such that (26) is violated. Clearly, satisfaction of all participation constraints implies that $\beta = \beta^0$. Consider the alternative credit contract (K^0, R_H^*, R_L^*) where

$$R_H^* = y_H - \beta - \frac{\bar{y}}{p(K^0)}$$

and

$$R_L^* = y_L.$$

Following the choice of the uniform rent β by the landlord, if the lender deviates by offering (K^0, R_H^*, R_L^*) , the seniority of his claims implies that the landlord obtains β as his payoff only when y_H is produced. When y_L is produced, the moneylender appropriates the entire output. Observe that, given the tenant's limited liability, it is evident that, in state L, she earns nothing, while she satisfies her participation constraint by earning exactly $\frac{\bar{y}}{p(K^0)}$ in state H. Furthermore, since (26) does not hold, and $\beta = \beta^0$, R_H^* is non-negative. Consequently, (K^0, R_H^*, R_L^*) constitutes a feasible and strictly profitable deviation by the lender.

Thus, if (26) is violated, the landlord's equilibrium payoffs cannot be uniform across the states of nature. A similar procedure leads to the same conclusion with respect to the violation of condition (27). It is easy to see that if β^0 is strictly less than y_L , and satisfies (26) and (27), it will be the optimal choice of the landlord, and that this is the only situation where the landlord earns uniform payoffs and the equilibrium outcome is first best. If, instead, the magnitude of β^0 is such that either $\beta^0 = y_L$, or any of the conditions (26) and (27) are violated, interlinkage will retain its superiority.

When the landlord is the second-mover, but possesses seniority of claims, the equilibrium

outcome may be first best even if the moneylender's payoffs are non-uniform. Consider, first, the case where $\bar{\beta} < \bar{y}$. It will be feasible and optimal for the moneylender to offer the contract $\{K^0, R_H^0, R_L^0\}$, where K^0 satisfies (3), and

$$R_L^0 = \max\left\{y_L - \frac{\bar{y} - s}{1 - p(K^0)}, 0\right\} \quad (28)$$

together with

$$R_H^0 = y_H - \frac{\bar{\beta} + \max\{\bar{y} - (1 - p(K^0))y_L, s\}}{p(K^0)} \quad (29)$$

for some $s \in (0, \bar{y} - \bar{\beta})$. Note that $\{[1 - \pi(K^0)]y_L - (\bar{y} - s)\}$ is maximized when $s = \bar{y} - \bar{\beta}$.

Hence, if $(1 - \pi(K^0))y_L = \bar{\beta}$, the moneylender can obtain the first-best optimum by choosing $R_L^0 = 0$, and $R_H^0 = y_H - \frac{\bar{\beta} + \bar{y} - (1 - p(K^0))y_L}{p(K^0)}$, and will extract the entire surplus by doing so. If, on the

other hand, $(1 - \pi(K^0))y_L > \bar{\beta}$, there exists a continuum of equilibria, one for each $s \in (s_0, \bar{y} - \bar{\beta})$, where $s_0 = \max\{\bar{y} - (1 - \pi(K^0))y_L, 0\}$. In each equilibrium in the continuum, the moneylender earns

a strictly positive payoff in both states, as given by the appropriate forms of (28) and (29). It remains the case, however, that K^0 is the optimal amount loaned, and that the lender extracts all surplus. If $\bar{y} > \bar{\beta}$, therefore, interlinked contracts cease to be strictly superior.

In the case where $\bar{\beta} \geq \bar{y}$, the moneylender's optimum leads to a first best outcome only if

$\bar{\beta} + \bar{y} > (1 - \pi(K^0))y_L$. Let the loan contract specify

$$R_L^0 = \begin{cases} \max\{y_L - \frac{\bar{y} - s}{1 - p(K^0)}, 0\} & \text{if } (1 - p(K^0))y_L \leq \bar{\beta} \\ 0 & \text{if } (1 - p(K^0))y_L > \bar{\beta} \end{cases} \quad (30a)$$

$$R_H^0 = y_H - \frac{\bar{\beta} + \bar{y} - (1 - p(K^0))y_L}{p(K^0)} \quad (30b)$$

together with K^0 as the optimal amount of the loan. If $[1 - \pi(K^0)]y_L > \bar{\beta}$, the lender's expected payoff is

$$\pi(K^0)y_H + [1 - \pi(K^0)]y_L - \bar{\beta} - \bar{y},$$

and he extracts the whole surplus. If, on the other hand, $[1 - \pi(K^0)]y_L = \bar{\beta}$, the lender can still extract the entire surplus, although it turns out that his payoff in state L is still zero, even if R_L^0 is strictly positive. This is because $(1 - \pi(K^0))y_L = \bar{\beta}$ and the participation constraint

$$\pi(K^0)y_H + [1 - \pi(K^0)]y_L > \bar{\beta} + \bar{y}$$

imply that $\pi(K^0)(y_H - R_H^0) = \bar{y}$, in which case, the choices $\beta_L = y_L$ and $\beta_H = \frac{\bar{\beta} - (1 - p(K^0))y_L}{p(K^0)}$

constitute the best response of the landlord. Nevertheless, when $\bar{y} < (1 - \pi(K^0))y_L$, inspection of (30a) reveals that the loan contract specifies a strictly positive R_L^0 . Otherwise, if the lender were to specify $R_L^0 = 0$, the landlord would charge the non-negative state-contingent rents $\beta_H = y_H$, and $\beta_L = y_L - \frac{\bar{y}}{1 - p(K^0)}$, to the disadvantage of the moneylender.

The remaining case is that where $\bar{\beta} \geq \bar{y}$, and $(1 - \pi(K^0))y_L = \bar{\beta} + \bar{y}$. Under these conditions,

it is easy to show that the moneylender is unable to design a contract that extracts all the surplus, and that K^0 is no longer optimal for him. Consequently, the equilibrium outcome of the sequential game where the moneylender is the first mover, and the landlord possesses seniority of claims, will be strictly inferior to that with interlinked contracts.

6.2 Multiple Instruments

Suppose now that each of the two principals controls a continuously variable instrument that affects expected yield at the margin. In particular, in addition to the moneylender supplying K , let a ($= 0$) denote a variable input, with price normalized to one, that is supplied by the landlord. It has an increasing effect on expected output in the following manner: $\pi = \pi(K, a)$ is increasing, concave and smooth in its arguments, with $\pi(K, 0) = \pi(0, a) = 0$. We shall now demonstrate that interlinkage is strictly superior, and that this superiority is immune to the order of moves, or the assignment of seniority of claims, in the game with two distinct principals.

Consider, first, the optimal interlinked contract. The landlord offering this contract specifies K^0 , a^0 , β_H and β_L to achieve the first-best solution, subject to the participation of the tenant. This implies that K^0 and a^0 maximize

$$\pi(K, a)y_H + (1 - \pi(K, a))y_L - (1 + m)K - a \quad (31)$$

and that β_H and β_L satisfy

$$\pi(K^0, a^0)[y_H - \beta_H] + (1 - \pi(K^0, a^0))[y_L - \beta_L] - (1 + m)K^0 - a^0 = \bar{y} \quad (32)$$

The first-order conditions that yield K^0 and a^0 as interior solutions are, respectively

$$\pi_K(K^0, a^0)[y_H - y_L] - (1 + m) = 0 \quad (33)$$

and,

$$\pi_e(K^0, a^0)[y_H - y_L] - 1 = 0 \quad (34)$$

If non-interlinked contracts are to specify the same K and a , it is evident that the principal who moves first must have the same payoffs in both states of nature. But then, the same principal would have no incentive to provide a positive amount of the costly instrument under his control. Consequently, the equilibrium outcome of this game would differ from that determined by the equations (32) – (34).

6.3 Summary

What are the implications that emerge from the analysis of the above subsections? First of all, we conclude that with limited liability and non-collusive principals, allocative efficiency is guaranteed only if a single principal has direct control of all instruments that affect production decisions at the margin, and, in addition, possesses seniority of claims. Second, if a “passive” principal – one whose decisions are limited to exercising his property rights to determine his share of the returns – is the first-mover, then allocative efficiency is impaired unless his equilibrium payoffs are uniform across states of nature. Uniform payoffs for the first-mover are, however, not essential for allocative efficiency if he is the only principal possessing a continuously variable instrument of control. As demonstrated above, apart from instances where $\bar{\beta}$ and $(1 - \pi(K^0))y_L$ were “too large”, the first best outcome was achieved with the first-mover earning payoffs that varied across the states of nature.

Finally, a multiplicity of continuously variable instruments, with at least one under the control of each principal, results in the unambiguous superiority of interlinkage, irrespective of the assignment of seniority of claims and the order of moves.

7. Conclusion

As noted in the introduction, if a landlord can write nonlinear contracts over a sufficiently large set of observables, and certain other conditions are fulfilled, he will gain nothing by writing contracts which interlink credit and land markets (Ray and Sengupta, 1989). So this route to explaining interlinkage seems, at first blush, to be blocked. If, however, the landlord cannot observe the terms set by a lender, our analysis of the case where the tenant enjoys limited liability is robust to any specification of the loan contract (non-linear or otherwise) that maximizes the residual surplus of the independent lender, and so establishes circumstances under which interlinking the credit and land contracts may lead to superior outcomes even if each contract can be non-linearly specified.

With the rise of new kinds of analysis, such as multi-principal models (Dixit; 1996, 1997), we expect that there will be a revival in explaining interlinkage as a form of collusive behavior among principals who were otherwise engaged in employing Nash equilibrium strategies. While some features of these models (such as multiple principals, risk-aversion and moral hazard) were also present in this paper, it is important to note that our analysis differs from the multi-principal literature in two aspects: our agent, the tenant, performs only one task, and the division of the (gross) returns from this task is the outcome of a game between the two principals. Thus, though the present paper belongs to the genre of multi-principal models in a broad sense, there are important idiosyncrasies in

the structure of our model, and the specific nature and the objective of our analysis differs in many details.

In our model, the axiom of limited liability is necessary to generate negative externalities in the absence of collusion between principals. It is this property which, under certain conditions, makes the landlord charge non-uniform rents, whereupon the outcome of the non-cooperative Nash game suffers from production inefficiency compared to the “collusive” alternative of interlinkage. Such inefficiency may occur even when the agent is risk-neutral. At one level, a limited liability clause may be thought of as one which sets limits to the range of nonlinear contracts that are feasible. Viewed in this manner, what our main result demonstrated was that within the bounds of limited liability, all externalities cannot be internalized and so a landlord can do better by usurping the role of the moneylender as well, thereby giving rise to interlinkage.

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APPENDIX

Proof of Proposition 1, part (i):

We demonstrate that $\beta_L = \min\{\beta^\circ, y_L\}$ is the optimal choice of the landowner in the equilibrium of the sequential game analyzed in section 4. As discussed in that section, $\beta_H = \beta_L = \beta^\circ$ if $\beta^\circ \leq y_L$. Consider the case where $\beta^\circ > y_L$, and assume, to the contrary that β_L is strictly less than y_L . Note that, with $\beta_H = y_H$, the moneylender maximizes his expected returns by choosing $K = 0$, so that the tenant will not participate in production. This implies that β_H must be strictly less than y_H in the equilibrium of the sequential game. Then, the participation constraint of the moneylender is strictly binding, as otherwise it is obvious from (7) that the landlord can increase expected profits, while avoiding any changes in K^m , by increasing both state-contingent rents by appropriately small and equal amounts. Thus, if $\beta_L < y_L$, with (6) representing the payoff of the moneylender, it follows that

$$p(K_1^m)\beta_H + (1 - p(K_1^m))\beta_L = p(K_1^m)y_H + (1 - p(K_1^m))y_L - (1 + m)K_1^m - \bar{y} \quad (\text{A.1})$$

where K_1^m denotes the optimal amount of finance advanced by the moneylender in this case.

Now consider the feasibility and optimality of the landlord choosing the pair of state contingent rents

$$\beta_H' = y_H - \frac{1}{p(K_1^m)} [(1 + m)K_1^m + \bar{y}] = \beta_H - \frac{1 - p(K_1^m)}{p(K_1^m)} (y_L - \beta_L) \quad (\text{A.2})$$

and,

$$\beta_L' = y_L \quad (\text{A.3})$$

It is easy to check that these rents satisfy the moneylender's and tenant's participation constraints as

strict equalities if the previous decisions of the moneylender (K_1^m and the corresponding interest rate) remain unchanged. If K_2^m is the optimal amount of funds loaned by the moneylender under the new circumstances, he can do no worse than before while still satisfying the participation constraint of the tenant. In addition, since the hypothesis $\beta_L < y_L$ implies that $\beta_H' < \beta_H$ and $\beta_L' > \beta_L$, it follows from (7) that K_2^m is strictly greater than K_1^m .

Denote $p(K_i^m)$ as p_i for $i = 1, 2$. With the new rents, the expected payoff of the landlord is, using (A.2) and (A.3),

$$p_2\beta_H' + (1 - p_2)\beta_L' = p_2y_H + (1 - p_2)y_L - \frac{p_2}{p_1}[(1 + m)K_1^m + \bar{y}] \quad (\text{A.4})$$

With the R.H.S. of (A.1) representing his payoff under the rents (β_H, β_L) , we have

$$[p_2\beta_H' + (1 - p_2)\beta_L'] - [p_2\beta_H + (1 - p_2)\beta_L] = \frac{p_2 - p_1}{p_1} [p_1(y_H - y_L) - (1 + m)K_1^m - \bar{y}] \quad (\text{A.5})$$

Again, using (A.1), it is easy to show that

$$p_1(y_H - y_L) - (1 + m)K_1^m - \bar{y} = p_2\beta_H + (1 - p_2)\beta_L - y_L \quad (\text{A.6})$$

If β_H and β_L are optimal rents, the R.H.S. of (A.6) must be non-negative, since, with $\beta^0 > y_L$, a uniform rent of y_L is feasible for the landlord, and results in a strictly positive payoff to the moneylender. Furthermore, as demonstrated in the proof of Proposition 1 in section 3, a small increase in β_H from the uniform rent of y_L is feasible, and would result in an expected payoff to the landlord that is strictly greater than y_L . Thus, for optimal rents, the R.H.S. of (A.6) is strictly positive.

But then, with $p_2 > p_1$, $p_2\beta_H' + (1 - p_2)\beta_L'$ is strictly greater than $p_2\beta_H + (1 - p_2)\beta_L$. This is a contradiction which cannot be resolved as long as $\beta_L < y_L < \beta^0$. Therefore, if β^0 is strictly greater than y_L , then $\beta_L = y_L$, and $\beta_H > y_L$.